11. Upwind:

$$T_i^{n+1} = T_i^n + C\left(T_{i-1}^n - T_i^n\right) + r\left(T_{i-1}^n + T_{i+1}^n - 2T_i^n\right) \qquad (1)$$

Consistencia:

Expandimos por Taylor:

- $T_i^{n+1} = T_i^n + \Delta t \frac{\partial T}{\partial t} + \frac{\Delta t^2}{2}\frac{\partial^2 T}{\partial t^2} + O(\Delta t^3)$
- $T_{i-1}^n = T_i^n - \Delta x \frac{\partial T}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 T}{\partial x^2} + O(\Delta x^3)$
- $T_{i+1}^n = T_i^n + \Delta x \frac{\partial T}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 T}{\partial x^2} + O(\Delta x^3)$

Sustituimos en: $\frac{T_i^{n+1} - T_i^n}{\Delta t} + u\frac{T_i^n - T_{i-1}^n}{\Delta x} - \alpha\frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} = 0$

$$\frac{\partial T}{\partial t} + \frac{\Delta t}{2}\frac{\partial^2 T}{\partial t^2} + \dots + u\left(\frac{\partial T}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 T}{\partial x^2} + \dots\right) - \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\Delta x^2}{12}\frac{\partial^4 T}{\partial x^4} + \dots\right) = 0$$

$$\Rightarrow \underbrace{\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} - \alpha\frac{\partial^2 T}{\partial x^2}\right)}_{\text{Ecuación de Transporte}} + \underbrace{\left[\frac{\Delta t}{2}\frac{\partial^2 T}{\partial t^2} - \frac{u\Delta x}{2}\frac{\partial^2 T}{\partial x^2}\right]}_{\text{Error}} = 0$$

El error tiende a cero si $\Delta t \to 0$ y $\Delta x \to 0$. Consistente de primer orden en espacio y tiempo.

Estabilidad:

Consideramos $T_i^{n+1} = \xi_i^n = G^n e^{j\theta i}$ e introducimos en la ecuación (1):

$$G^{n+1} e^{j\theta i} = G^n e^{j\theta i} + C G^n \left(e^{j\theta(i-1)} - e^{j\theta i} \right) + \lambda G^n \left(e^{j\theta(i-1)} + e^{j\theta(i+1)} - 2 e^{j\theta i} \right)$$

$$G = 1 + C \left(e^{-j\theta} - 1 \right) + \lambda \left(e^{j\theta} + e^{-j\theta} - 2 \right)$$

$$= 1 + C \cos\theta - C - j C \operatorname{sen}\theta + 2\lambda \left(\cos\theta - 1 \right)$$

$$= -1 + \cos\theta \left(2\lambda + C \right) - C - j C \operatorname{sen}\theta$$

tal que es estable si $|G| \leq 1$:

$$|G| = \sqrt{C^2 \operatorname{sen}^2\theta + \left(\cos\theta (2\lambda + C) - C - 1 \right)^2} \leq 1$$